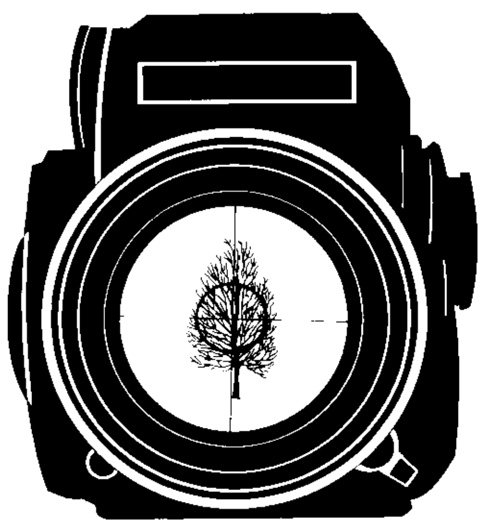
Module 659

Modules in Undergraduate Mathematics and its Applications

The Mathematics of Focusing a Camera

Raymond N. Greenwell



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INTERMODULAR DESCRIPTION SHEET:	UMAP Unit 659
TITLE:	THE MATHEMATICS OF FOCUSING A CAMERA
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CLASSIFICATION:	Applications of mathematics to photography.
ABSTRACT:	This unit introduces the reader to several photographic concepts, including focal length, f-stop, and depth of field. Using intermediate algebra and trigonometry, we derive the formulas listed in the Kodak Customer Service Pamphlet on the subject. We also show how these formulas can be used in photography.
PREREQUISITES:	Intermediate algebra and trigonometry.

The Mathematics of Focusing a Camera

by

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Modules and Monographs in Undergraduate Mathematics and its Applications Project (UMAP)

The goal of UMAP was to develop, through a community of users and developers, a system of instructional modules in undergraduate mathematics and its applications to be used to supplement existing courses and from which complete courses may eventually be built.

The Project was guided by a National Advisory Board of mathematicians, scientists, and educators. UMAP was funded by a grant from the National Science Foundation and is now supported by the Consortium for Mathematics and Its Applications, Inc. (COMAP), a nonprofit corporation engaged in research and development in mathematics education.

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1. Introduction

Before the spring of 1982, my only camera was an Instamatic, a simple device that I recklessly dragged through the mud on my backpacking trips. Then I bought my first 35mm camera. I was amazed at the number of buttons and dials. Once I got the box open, the camera inside seemed even more complex. I soon learned to take pictures I could never have taken with an Instamatic, such as an inspiring close-up of the inside of the lens cover.

Another item I obtained at the same time was the Kodak Customer Service Pamphlet AA-26, entitled *Optical Formulas and Their Applications.* Here was something of interest to a mathematician. Included was not only the fundamental lens equation, sometimes called the thin lens equation, but also others involving focal length, magnification, and depth of field. In this module we shall derive those equations and present some of their applications.

2. Focal Length

The first term we need is the *focal length* of the camera lens, which we will denote F. If parallel rays of light approach the lens, they are focused at a point, as shown in Fig. 1. The distance from this point to the lens is F. If light rays from a point infinitely far away approach the lens, they would be parallel, as in Fig. 1. For all practical purposes, infinity isn't very far away. On a standard camera lens (one with a focal length of about 50 mm) we can consider anything more than 20 m away to be at infinity. (If you are unfamiliar with the metric system, see Section 10.)

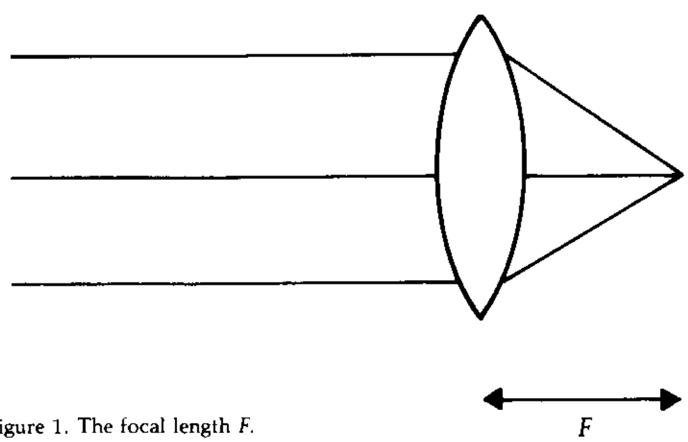

Figure 1. The focal length F.

The easiest way to find the focal length of a lens is to read it on the front or side of the lens. The lens that came with my camera has a focal length of 50 mm. The wide angle lens I bought has $F = 28$ mm, while the zoom lens has an F that varies from 80 mm to 200 mm. We shall discuss later the effect of the focal length on the pictures you take.

3. F-Stops

The *f-stop*, or *f-number*, of a lens is the focal length divided by the diameter of the lens opening, which we will denote by L. Mathematically, this says

$$f = \frac{F}{L}. \tag{1}$$

The f-stop is written as **f** followed by a slash followed by the numerical value, such as **f/8**. For a given focal length F, the f-stop is made smaller by making the lens opening, known as the *aperture*, larger. This increases the amount of light that falls on the film, which is what we want under low light conditions.

Unfortunately, as we shall see later, increasing the aperture also tends to put more of the picture out of focus. This might be desirable if we want to focus attention on one part of the picture and put everything else out of focus. Otherwise, we can let in more light by using a slower shutter speed, which causes the aperture to be open for a longer period of time. The disadvantage of this is that any movement of the camera or subject results in a blurry picture. Another solution is to use film with a higher ASA (or ISO) number, which means it requires less light. The drawback is that the pictures will then have a grainier texture. In photography, as in life, there are no free lunches.

Most lenses have the following f-stops: **f/2**, **f/2.8**, **f/4**, **f/5.6**, **f/8**, **f/11**, **f/16**, and **f/22**. These are the full stops, but there are also half stops in between these. The full stops are set so that each lets in half as much light as the previous stop. The amount of light let in is proportional to the area of the aperture, which is roughly circular, and the area of a circle is proportional to the diameter squared. ($A = \pi r^2 = \pi L^2/4$, where $L = 2r$.) Thus, the diameter must be reduced by $1/\sqrt{2}$ to reduce the area by

1/2. According to Eq. (1), if L is multiplied by $1/\sqrt{2}$, then f is multiplied by $\sqrt{2}$. For example, if the first f-stop is 2, the next one is $2\sqrt{2} = 2.828$, which is rounded to 2.8. We can continue this process and compute the exact values of all the full stops.

Exercise

1. Find the exact value of all the full f-stops. Which two of the values given previously are rounded incorrectly?

4. The Fundamental Lens Equation

We shall now derive the fundamental lens equation. Let u be the distance from the object focused upon to the lens, and let v be the distance from the lens to the film. The light rays travel through the paths indicated in Fig. 2. We denote the height of the object by h and the height of the image on the film by h'. If u approaches infinity, F and v become equal. (Recall the definition of F in Section 2.) Otherwise, the lens must be farther from the film than F, and this extra distance is denoted x. This is why the lens housing becomes longer when you focus on a close object.

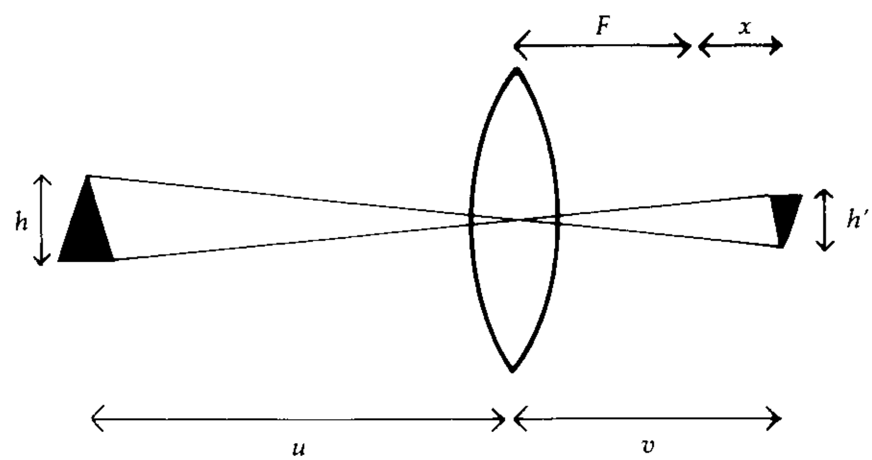

Figure 2. Quantities involved in the fundamental lens equation.

Now let us focus our attention at a point on the top of the object denoted P (see Fig. 3). The light rays from this point do not all travel straight to the point at the bottom of the image, denoted P'. They travel out in all directions, and all rays from P that hit the lens are focused on P'. Two such rays are shown in Fig. 3, one traveling horizontally to point A, and the other traveling to a point level with P', which we call B. Because parallel rays of light are focused at a distance F from the lens, the points C and C' must both be a distance F from the lens. If we denote the point in the middle of the lens by D, then $\overline{CD} = \overline{C'D} = F$.

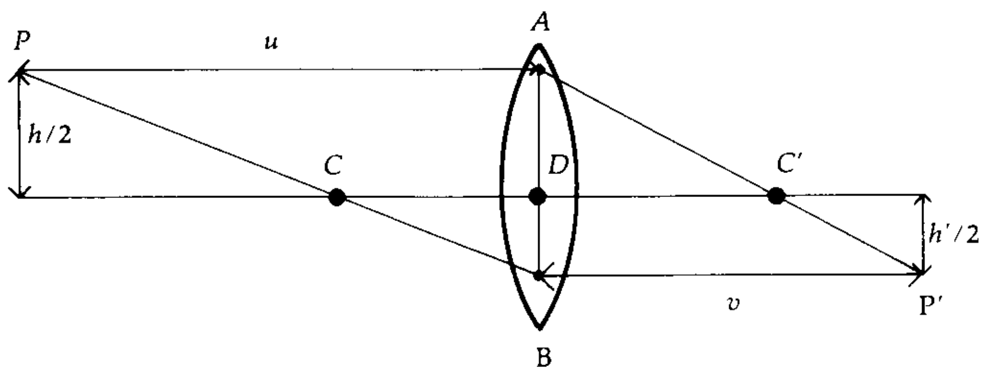

Figure 3. Rays of light focused by the lens.

Since triangles PAB and CDB are similar, we have

$$\frac{\overline{AB}}{\overline{PA}} = \frac{\overline{DB}}{\overline{CD}}. \tag{2}$$

Recalling that $\overline{PA} = u$, and seeing from Fig. 3 that

$$\overline{AB} = h/2 + h'/2,$$

Eq. (2) is transformed into

$$\frac{h/2 + h'/2}{u} = \frac{h'/2}{F}. \tag{3}$$

Next, we note that triangle ABP' and ADC' are similar, so we have

$$\frac{\overline{AB}}{\overline{PB}} = \frac{\overline{AD}}{\overline{C'D}}, \tag{4}$$

which becomes

$$\frac{h/2 + h'/2}{v} = \frac{h/2}{F}. \quad (5)$$

Adding Eqs. (3) and (5) gives

$$\frac{h/2 + h'/2}{u} = \frac{h/2 + h'/2}{v} = \frac{h'/2}{F} + \frac{h/2}{F}. \quad (6)$$

Dividing by $h/2 + h'/2$ yields

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{F}. \quad (7)$$

This is the important formula we have been seeking.

Exercises

2. If a 50 mm lens is focused on an object 5 m away, how far is the lens from the film?

3. We earlier stated that if $u \geq 20$ m for a 50 mm lens, the object is considered to be at infinity, so v and F are essentially equal. Find v for a 50 mm lens if $u = 20$ m.

5. Other Forms of The Lens Equation

The Kodak booklet has several formulas immediately after the lens equation that are described as "more directly useful." They first define the magnification m:

$$m = \frac{h'}{h}. \quad (8)$$

If the image on the film is equal in size to the object being photographed, then $m = 1$. In most cases the image on the film is much smaller, so $m < 1$.

From the similarity of triangles in Fig. 2, we see that we can also express the magnification as

$$m = \frac{v}{u}. \quad (9)$$

Exercises

4. Derive the equations

$$u = \frac{Fv}{v - F} \tag{10}$$

and

$$v = \frac{Fu}{u - F}. \tag{11}$$

5. Use Eq. (10) to eliminate u from Eq. (9) and get

$$m = \frac{v - F}{F}. \tag{12}$$

6. Use Eq. (11) to eliminate v from Eq. (9) and get

$$m = \frac{F}{u - F}. \tag{13}$$

Notice from Eq. (13) that m gets smaller as u gets larger. If an object really could be infinitely far away, then the magnification would be zero. For example, if you photograph a golf ball from very far away, the image on the film virtually disappears.

We can draw an important conclusion from Eq. (13). If F is made larger, the numerator of the right-hand side increases while the denominator decreases, resulting in a larger value of m. In other words, a lens with a large focal length has a large magnification. If I want to photograph a moose that is far away, I use my telephoto lens with $F = 200$ mm so that the animal will have a large image in the picture. On the other hand, if I want to photograph the Grand Canyon, I use my wide angle lens with $F = 28$ mm, and the small magnification allows more of the Grand Canyon to fit my picture.

Now let's derive another useful equation. If we solve Eq. (9) for v, we get $v = mu$. Putting this into Eq. (7) gives

$$\frac{1}{mu} + \frac{1}{u} = \frac{1}{F}$$

or

$$\frac{1}{u}\left(\frac{1}{m} + 1\right) = \frac{1}{F}.$$

Multiplying by uF yields

$$u = F\left(\frac{1}{m} + 1\right) . \tag{14}$$

As an example of how we can use Eq. (14), suppose we want to photograph a 180 cm person with a 50 mm lens so that the image completely fills up the picture the long way. Since 35 mm film has an image 24 mm by 36 mm, the magnification is 36 mm/180 cm = 0.02. Putting this and $F = 50$ mm = 0.05 m into Eq. (13) yields $u = 2.55$ m. Thus our subject must stand 2.55 m away.

Exercises

7. Find out how far away our subject must stand if we want him to completely fill up the image the short way and we use our wide angle lens, with a focal length of 28 mm.

8. What focal length should we use to photograph the Empire State Building, which is 381 m high, if we are standing 200 m away? Assume that we want the building to almost fill up the picture the long way, with 2 mm of extra space at the top and bottom of the negative.

9. The Kodak booklet also lists the following formulas. Derive each of them:

   a) $v = (m + 1)F$

   b) $u = \left(\frac{1}{m} + 1\right)F$

   c) $u + v = \frac{(m + 1)^2}{m} F$

We can also compute x, the distance the lens must move from the infinity setting. From Fig. 1, we see that

$$x = v - F,$$

and using Eq. (11) gives

$$x = \frac{Fu}{u - F} - F\,\frac{(u - F)}{(u - F)} = \frac{F^2}{u - F} . \tag{15}$$

10. How much must a 50 mm lens move from the infinity position to focus on an object 1.5 m away?

6. Depth of Field

When the camera is focused at a given distance, points not too much nearer or farther away are still roughly in focus. The difference between the nearest and farthest points from the camera that are still in focus is called the *depth of field*. The exact location of those two points, called the near and far limits of the depth of field, is a matter of opinion, since objects do not suddenly become completely out of focus when they reach the limits of the depth of field. Furthermore, what is considered in focus for a 3 x 5 inch print may not be acceptable for an 8 x 12 inch enlargement.

General standards for the depth of field depend upon something called the circle of confusion. This is not one of those perplexing traffic circles in Washington, D.C. The circle of confusion is the circular image on the film of a point not exactly in focus. If the circle is small enough, the point appears to be in focus, and is therefore in the depth of field. Fig. 4 shows why a point can result in a circle, rather than a point, on the film. Point P is exactly in focus, resulting in a point P' on the film. Point Q is closer than point P, and the lens focuses it at a point Q' behind the film. The film receives a circle of diameter d; it looks like a line segment in Fig. 4 because we are viewing it from the side. A similar event occurs if Q is further from the lens than P, as shown in Fig. 5. According to the Kodak booklet, the most widely used value of d for 35 mm film is 0.002. (Some books recommend a smaller value.) We will derive formulas for the near and far limits of the depth of field as a function of d.

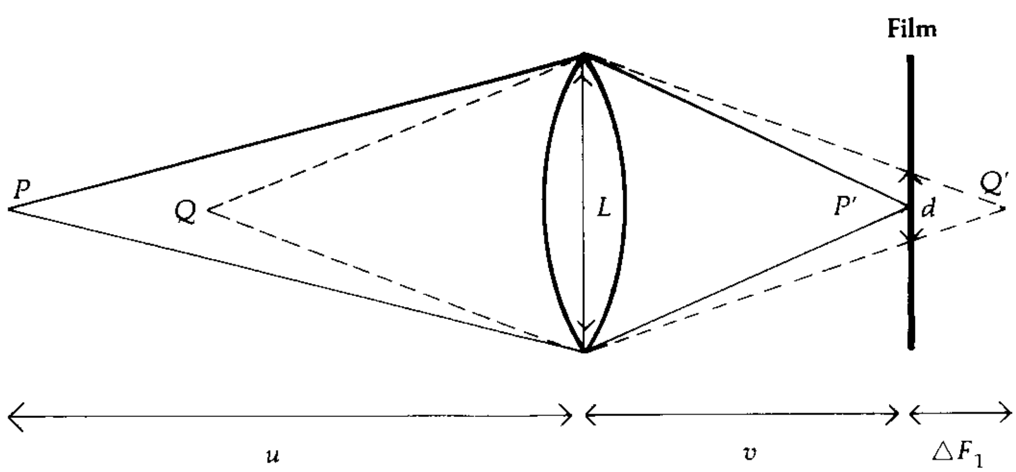

Figure 4. Origin of the circle of confusion.

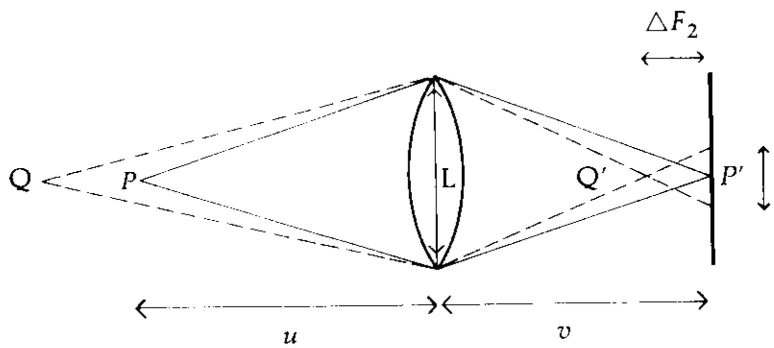

Figure 5. The circle of confusion for a point farther away.

Let $\triangle F_1$ be the distance from P' to Q', as shown in Fig. 4. Then by similarity of the triangle with Q' as a vertex and L as a base to the triangle with Q' as vertex and d as a base, we have

$$\frac{\triangle F_1}{d} = \frac{v + \triangle F_1}{L}. \tag{16}$$

The quantity $\triangle F_1$ is very small compared with v, so we will drop it. Recall from Fig. 2 that $v = F + x$, so Equation (16) becomes

$$\frac{\triangle F_1}{d} = \frac{F + x}{L}. \tag{17}$$

We now use similarity of the two triangles in Fig. 5 with Q' as a vertex, one with L as a base and the other with d as a base, to get

$$\frac{\triangle F_2}{d} = \frac{v - \triangle F_2}{L}. \tag{18}$$

and by the same reasoning as before, this becomes

$$\frac{\triangle F_2}{d} = \frac{F + x}{L}. \tag{19}$$

Comparing Eqs. (17) and (19), we see that $\triangle F_1$ and $\triangle F_2$ are equal, at least to the degree of our approximation, so we will denote them both by $\triangle F$. Either equation tells us

$$\triangle F = \frac{d}{L}(F + x). \tag{20}$$

Let u_1 be the near limit of the depth of field, corresponding to the distance of point Q from the lens in Fig. 4. Let u_2 be the far limit of the depth of field, corresponding to the distance of point Q from the lens in Fig. 5. Equation (10) gives us a formula to compute these two quantities. To get u_1, we substitute $v + \triangle F$ for v in Eq. (10), and then let $v = F + x$, yielding

$$\begin{aligned} u_1 &= \frac{F(v + \triangle F)}{(v + \triangle F) - F} \\ &= \frac{F(F + x + \triangle F)}{F + x + \triangle F - F} \\ &= \frac{F(F + x + \triangle F)}{x + \triangle F}. \end{aligned} \tag{21}$$

As before, the $\triangle F$ in the numerator is small compared with $F + x$, so we drop it. We do not drop it in the denominator, since x is not very big. Thus Eq. (21) becomes

$$u_1 = \frac{F(F + x)}{x + \triangle F}. \tag{22}$$

Exercise

11. Use reasoning similar to the reasoning given above to derive the equation

$$u_2 = \frac{F(F + x)}{x - \triangle F}. \tag{23}$$

We would like to get rid of x and $\triangle F$, whose values we usually do not know, and write Eqs. (22) and (23) in terms of more familiar quantities. To do this, we now introduce one more new concept, the *hyperfocal distance*. Denoted by H, the hyperfocal distance is just the near limit of the depth of field when the lens is focused at infinity. We saw earlier that when u becomes infinite, x becomes 0. Then Eq. (22) tells us that the near limit of the depth of field is

$$H = \frac{F^2}{\triangle F}. \tag{24}$$

We can eliminate $\triangle F$ by using Eq. (20), with $x = 0$, giving

$$H = \frac{F^2}{(d/L)F} = \frac{FL}{d}. \tag{25}$$

A better equation for H does not use L, which we have to compute, and instead uses f, which we can read off the side of the lens. We can use Eq. (11) to change from L to f, yielding

$$H = \frac{F(F/f)}{d} = \frac{F^2}{fd}. \tag{26}$$

We will now eliminate $\triangle F$ from Eq. (22) by using Eq. (24), which tells us that

$$\triangle F = \frac{F^2}{H}. \tag{27}$$

Putting this into Eq. (22) gives

$$\begin{aligned} u_1 &= \frac{F(F + x)}{x + F^2/H} \\ &= \frac{HF(F + x)}{Hx + F^2}. \end{aligned} \tag{28}$$

Finally, we need an equation to eliminate x. Looking back over the multitude of equations we have derived, we see that Eq. (15) will do the job. The result is

$$\begin{aligned} u_1 &= \frac{HF\left(F + \dfrac{F^2}{u - F}\right)}{H\left(\dfrac{F^2}{u - F}\right) + F^2} \times \frac{u - F}{u - F} \\ &= \frac{HF\,(F(u - F) + F^2)}{HF^2 + F^2(u - F)} \\ &= \frac{HF^2(u - F + F)}{F^2(H + u - F)} \end{aligned}$$

$$= \frac{Hu}{H + (u - F)}. \tag{29}$$

This is the equation for the near limit of the depth of field as given in the Kodak booklet.

Exercises

12. Following the procedure above, derive the equation for the far limit of the depth of field:

$$u_2 = \frac{Hu}{H - (u - F)}. \tag{30}$$

13. Letting $d = 0.002$ inches, find the hyperfocal distance of a 50 mm lens set at **f/8**.

14. Use your answer from Exercise 13 to find the near and far limits of the depth of field for a 50 mm lens set at **f/8** focused on a point 5 m away.

15. If I set my 50 mm lens at **f/8** and focus on 2 m, little numbers on the side indicate that the near and far limits of the depth of field are roughly 1.7 m and 2.6 m, respectively. What are the makers of my lens using for the diameter of the circle of confusion?

7. More Depth of Field

The Kodak booklet lists two additional formulas for the depth of field. They involve the angular size of the circle of confusion, denoted by θ in Fig. 6. The Kodak booklet suggests letting θ be 2 minutes of arc, or $1/30°$.

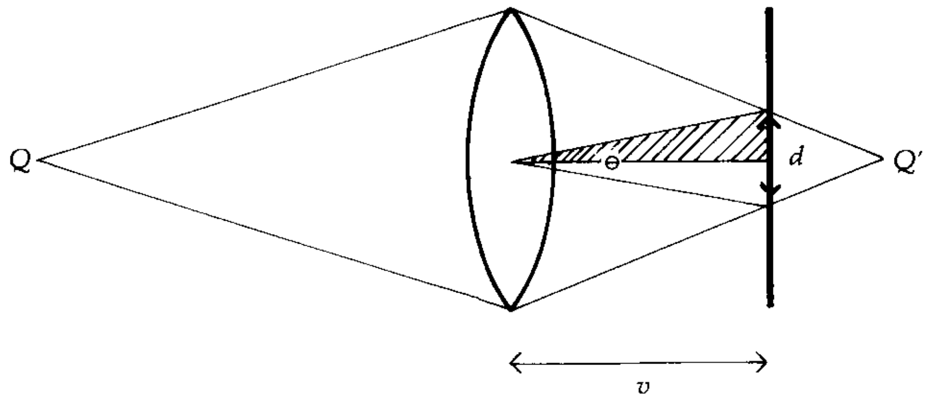

Figure 6. Angular size of the circle of confusion.

We can see from the shaded triangle that

$$\tan\frac{\theta}{2} = \frac{(d/2)}{v}. \tag{31}$$

It would be easier to write Eq. (31) in terms of the tan θ. Using the double angle formula for tangents, we can write

$$\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}. \tag{32}$$

But $\tan^2(\theta/2)$ is very small, compared with 1, for such a small θ, so Eq. (32) tells us that the equation

$$\tan\frac{\theta}{2} = \frac{1}{2}\tan\theta \tag{33}$$

is approximately true for small θ. Then Eq. (31) can be written

$$\tan\theta = \frac{d}{v}. \tag{34}$$

Using Eq. (11) to eliminate v from Equation (34),

$$\tan\theta = \frac{d(u-F)}{Fu}$$

or

$$u - F = \frac{Fu(\tan \theta)}{d}. \qquad (35)$$

We will need this in a few moments. First, we will note that the new formulas in the Kodak booklet give the near and far limits of the depth of field measured from the plane focused upon, rather than from the camera. (See Fig. 7.) Letting w_1 and w_2 be these quantities, we have

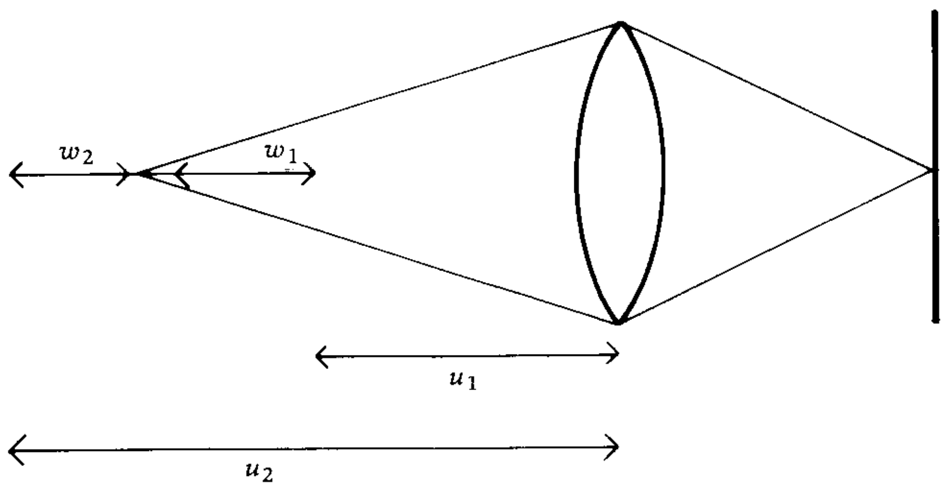

Figure 7. Measuring depth of field from plane focused upon.

$$\begin{aligned} w_1 &= u - u_1 \\ &= u - \frac{Hu}{H + u - F} \\ &= \frac{u(H + u - F) - Hu}{H + u - F} \\ &= \frac{u(u - F)}{H + (u - F)}. \qquad (36) \end{aligned}$$

Exercise

16. Derive the equation for the far limit of the depth of field as measure from the plane focused upon:

$$w_2 = \frac{u(u - F)}{H - (u - F)}. \qquad (37)$$

Now we can use Eq. (35) to eliminate $u - F$ from Eq. (36), yielding

$$w_1 = \frac{u \dfrac{Fu(\tan \theta)}{d}}{H + \dfrac{Fu(\tan \theta)}{d}}$$

$$= \frac{Fu^2(\tan \theta)}{Hd + Fu(\tan \theta)}. \tag{38}$$

Finally, we get rid of H by using Eq. (25) to write $Hd = FL$. Then Eq. (37) becomes

$$w_1 = \frac{Fu^2(\tan \theta)}{FL + Fu(\tan \theta)}$$

$$= \frac{u^2(\tan \theta)}{L + u(\tan \theta)}. \tag{39}$$

This is the formula in the Kodak booklet.

Exercises

17. Following the procedure above, derive the form for the far limit of the depth of field as measured from the plane focused upon:

$$w_2 = \frac{u^2(\tan \theta)}{L - u(\tan \theta)}. \tag{40}$$

18. Using $\theta = 1/30°$, find w_1 and w_2 for 50 mm lens set at **f/8** focused on a point 5 m away. How does this compare with your answer to Exercise 14?

8. Bibliography

The Kodak Customer Service Pamphlet AA-26, *Optical Formulas and Their Applications*, costs 10 cents and can be purchased at most photography stores or by writing to Eastman Kodak Company, Consumer Markets Division, 343 State Street, Rochester, New York 14650. The following books may also be of interest:

Adams, Ansel, *The Camera*, New York Graphic Society, 1980.

Cox, A., *Optics, the Techniques of Definition*, Focal Press, 1966.

Magill, Arthur A., "Still Cameras," in *Applied Optics and Optical Engineering Vol. IV, Optical Instruments*, edited by Rudolf Kingslake, Academic Press, 1967.

Neblette, C. B. and Murray, Allen E., *Photographic Lenses*, Morgan and Morgan, Inc., 1973.

9. Answers to Exercises

1. $2 \times (\sqrt{2})^2 = 4$, $2 \times (\sqrt{2})^3 = 5.66$, $2 \times (\sqrt{2})^4 = 8$, $2 \times (\sqrt{2})^5 = 11.31$, $2 \times (\sqrt{2})^6 = 16$, $2 \times (\sqrt{2})^7 = 22.62$. The third f-stop should be rounded to 5.7 rather than 5.6, and the last one should be rounded to 23 rather than 22.

2. 5 m = 5000 mm.
   $1/5000 + 1/v = 1/50 \Rightarrow 1/v = 1/50 - 1/5000 = 99/5000$,
   $v = 5000/99 = 50.505$ mm.

3. 20 mm = 20000 mm.
   $1/20000 + 1/v = 1/50 \quad 1/v = 1/50 - 1/20000 = 399/20000$, $v = 20000/399 = 50.125$ mm.

4. $1/u = 1/F - 1/v = (v - F)/vF$, so $u = vF/(v - F)$. Eq. (11) is derived similarly.

5. $$m = \frac{v}{u} = \frac{v}{\left(\frac{vf}{v - F}\right)} = \frac{v(v - F)}{vF} = \frac{v - F}{F}.$$

6. $m = \frac{v}{u} = \frac{\left(\frac{uF}{u - F}\right)}{u} = \frac{uF}{u(u - F)} = \frac{F}{u - F}.$

7. $m = 2.4 \text{ cm}/180 \text{ cm} = 0.01333.$

$u = 0.028\left(\frac{1}{0.01333} + 1\right) = 2.13 \text{ m}.$

8. $h' = 36 - 2 \times 2 = 32 \text{ mm} = 0.032 \text{ m}.$

$m = 0.032/381 = 8.399 \times 10^{-5}$

$= F/(200 - F) \quad 0.016798 - 8.399 \times 10^{-5}F = F,$

$0.016798 = F(1 + 8.399 \times 10^{-5}),$

$F = 0.017698/(1 + 8.399 \times 10^{-5}) = 0.017 \text{ m} = 17 \text{ mm}.$

9. **a)** From Eq. (12), $mF = v - F$, so $v = mF + F = (m + 1)F$.

b) From Eq. (13),
$mu - mF$, so $mu = F + mF$, $u = (F/m) + F = (1/m + 1)F$.

c) Adding the answers to (**a**) and (**b**),

$u + v = (m + 1)F + \left(\frac{1}{m} + 1\right)F$

$= \left(m + 2 + \frac{1}{m}\right)F = \left(\frac{m^2 + 2m + 1}{m}\right)F = \frac{(m + 1)^2}{m}F.$

10. $u = 1.5 \text{ m} = 1500 \text{ mm}.$ $x = \frac{50^2}{1500 - 50} = 1.7 \text{ mm}.$

11. $u_2 = \frac{F(v - \triangle F)}{(v - \triangle F)} = \frac{F(F + x - \triangle F)}{F + x - \triangle F - F}$

$= \frac{F(F + x - \triangle F)}{x - \triangle F} \cong \frac{F(F + x)}{x - \triangle F}$

12. $$u_2 = \frac{F(F + x)}{x - \frac{F^2}{H}} = \frac{HF(F + x)}{Hx - F^2} = \frac{HF(F + u - F)}{H(\frac{F^2}{u - F}) - F^2}$$

$$= \frac{HF(F(u - F) + F^2)}{HF^2 - F^2(u - F)} = \frac{HF^2(u - F + F)}{F^2(H - (u - F))} = \frac{Hu}{H - (u - F)}.$$

13. $d = 0.002 \text{ in} \times 25.4 \text{ mm/in} = 0.0508 \text{ mm}$.

$$H = \frac{50^2}{8 \times 0.0508} = 6152 \text{ mm} = 6.2 \text{ m}.$$

14. $$u_1 = \frac{6.2 \times 5}{6.2 + 5 - 0.05} = 2.8 \text{ m}.$$

$$u_2 = \frac{6.2 \times 5}{6.2 - (5 - 0.05)} = 25 \text{ m}.$$

15. Using Eq. (29), $1.7 = \frac{H \times 2}{H + 2 - 0.05}$ $\quad 1.7H + 3.315 = 2H,$

$0.3H = 3.315$, $H = 3.315/0.3 = 11$ m.

Checking this with Eq. (30),

$2.6 = \frac{H \times 2}{H - (2 - 0.05)}$ $\quad 2.6H - 5.07 = 2H$, $0.6H = 5.07$,

$H = 5.07/0.6 = 8.5$ m. Using the first value in Eq. (26),
$d = F^2/fH = 50^2/8 \times 11000 = 0.028/25.4 \text{ mm/in} = 0.001$ in.

Using the second value,
$d = 50^2/8 \times 8500 = 0.037 \text{ mm}/25.4 \text{ mm/in.} = 0.001$ in.

16. $$w_2 = u_2 - u = \frac{Hu}{H - (u - F)} - u = \frac{Hu - u(H - u + F)}{H - (u - F)}$$

$$= \frac{u(u - F)}{H - (u - F)}.$$

17. $$w_2 = \frac{u\dfrac{Fu(\tan\theta)}{d}}{H - \dfrac{Fu(\tan\theta)}{d}} = \frac{Fu^2(\tan\theta)}{Hd - Fu(\tan\theta)} = \frac{Fu^2(\tan\theta)}{FL - Fu(\tan\theta)}$$

$$= \frac{u^2(\tan\theta)}{L - u(\tan\theta)}.$$

18. $$L = \frac{0.05\ m}{8} = 0.00625\ \text{m}.$$

$$w_1 = \frac{5^2 \tan \dfrac{1^\circ}{30}}{0.00625 + 5 \times \tan \dfrac{1^\circ}{30}} = 1.6\ \text{m}.$$

$$w_2 = \frac{5^2 \tan \dfrac{1^\circ}{30}}{0.00625 - 5 \times \tan \dfrac{1^\circ}{30}} = 4.4\ \text{m}.$$

This corresponds to $u_1 = u - w_1 = 5 - 1.6\ \text{m} = 3.4\ \text{m}$ and $u_2 = u + w_2 = 5 + 4.4\ \text{m} = 9.9\ \text{m}$. This is much narrower than the depth of field in Exercise 14.

10. Appendix: The Metric System for Length

The standard unit of length in the metric system is the meter, which is 39.37 inches, or slightly more than a yard. Conversely, one inch is 1/39.37 meters, or 0.0254 meters. Therefore, to convert inches to meters, we multiply the quantity in inches by 0.0254 m/in. For example, 42 in = 42 × 0.0254 = 1.07 m (rounded off). To convert meters to inches, divide by 0.0254 m/in. For example, 2.1 m = 2.1/0.0254 = 83 in.

A meter is divided into 100 centimeters, each of which is divided into 10 millimeters. There are 1000 millimeters in a meter. It is easy to convert back and forth from millimeters to meters, as is frequently done in this unit. Just move the decimal point three places to the right to go from meters to millimeters, and three places to the left to go from millimeters to meters. For example, 50 mm = 0.050 m, and 1.6 m = 1600 mm.

Since there are 0.0254 meters in an inch, there are 25.4 mm in an inch. We can thus convert inches to millimeters by multiplying by 25.4 mm/in. For example, 12 in. = 12 × 25.4 = 305 mm, 50 mm = 50/25.4 = 1.97 in.